A Survey of the Theory of Wire Grids*

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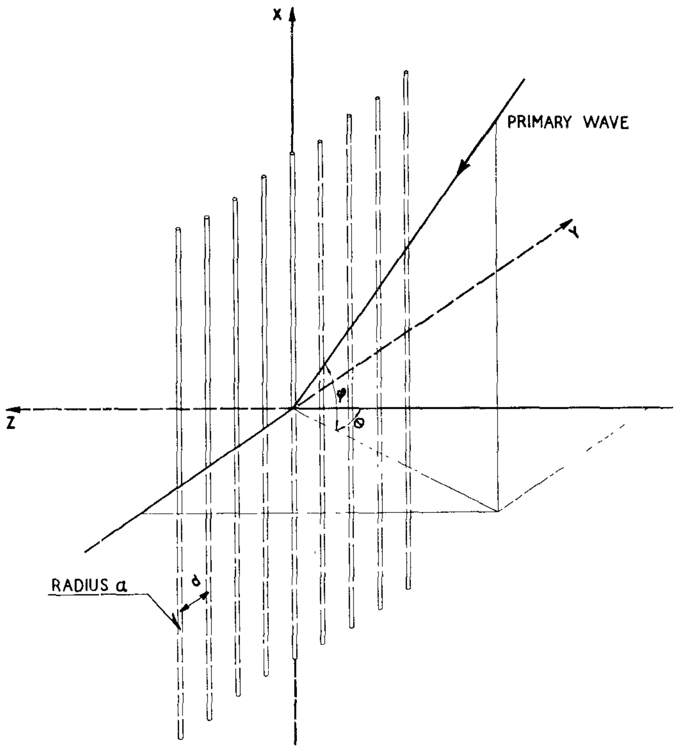
Summary—**The paper gives a survey of the literature concerning the electromagnetic properties of wire grids. As an introduction to the literature survey, a short description of the properties and applications of wire grids is given. Finally some particular grid configurations are mentioned.**

I. Introduction

THIS SURVEY is written from an electromagnetic point of view and all but a few papers concerning optical grids have been omitted. The survey does not claim to be complete. Papers, the contents of which are known by the author only from summaries in the *Electrical Engineering Abstract* or other abstracts are marked with an asterisk.

A description of the properties and applications of grids is given as an introduction to the literature survey. This is followed by the survey, which is ordered chronologically and divided into two parts: early papers, and currently used papers from a later period. Finally is given a description of some particular grid configurations.

Most attention is paid to the theoretical papers dealing with the simple grid with wires of circular cross section. The formulas found by the various authors for the reflection coefficient of such a gird are given and compared. For the rest of the papers describing experimental investigations and examinations of grids with noncircular wires and special grid configurations only brief descriptions are given, as the results in most of these cases cannot be stated briefly. The notation used in the formulas is shown in Fig. 1. The time factor is $e^{-i\omega t}$ and the Giorgi unit system is used.



	AIR	WIRES
DIELECTRIC CONSTANT	ε_0	$\varepsilon_1 = \varepsilon_r \varepsilon_0$
PERMEABILITY	μ_0	$\mu_1 = \mu_r \mu_0$
CONDUCTIVITY	$\sigma_0 = 0$	σ_1
PROPAGATION CONSTANT	$k_0 = \omega\sqrt{\varepsilon_0 \mu_0}$	$k = \omega\sqrt{\mu_1 \varepsilon_1 \left(1 + i\frac{\sigma_1}{\omega \varepsilon_1}\right)}$
INTRINSIC IMPEDANCE	$\zeta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$	$\zeta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1 \left(1 + i\frac{\sigma_1}{\omega \varepsilon_1}\right)}}$

Fig. 1—Plane grating with a plane wave at arbitrary incidence.

II. Properties and Applications of Grids

The grids which will be discussed in this paper may be defined as plane metallic systems which have in one direction a periodic structure with a period called the grid constant, which may be of the same order of magnitude as the wavelength of the incident field. The material properties of the grid are constant in the direction of the wires. The influence of such a grid depends primarily on the polarization of the incident wave and of the ratio between the grid constant and the wavelength. This dependence will be mentioned briefly in what follows together with the definition of the quantities commonly used to describe the properties of grids. Some applications of grids with a few references to papers describing these applications are also briefly mentioned.

A. Polarization

The grid usually has the greatest influence when the electric vector of the incident wave is polarized in the direction of the wires. This is called the Hertz effect; in certain cases, however, the grid may effect a wave polarized perpendicular to the wires more severely than a wave polarized parallel to the wires; this is called the Dubois effect. The type of transmission that takes place for a given grid depends upon the ratio between the wire diameter and the wavelength and upon the material of which the grid is made.

In radio engineering the Hertz effect is of most in-

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terest. It was exactly this effect which Hertz [1] utilized in 1889 in his experiments. Around the end of the century there were several theoretical investigations dealing with the electromagnetic fields near grids (see Section III-A). Formulas for the reflection coefficient of a grid were derived for both the case where the polarization of the incident wave is parallel and where the polarization is transverse to the wires. It turned out that usually the reflection coefficient was vanishingly small when the polarization of the incident wave was perpendicular to the wires. In fact, in most of the recent work on the subject (see Section III-B) it is assumed that a wave with a polarization perpendicular to the wires of the grid passes the grid uninfluenced. In computing the field around a grid one often expresses the total field as the incident field as it would be if the grid were not present plus a scattered field radiated from the grid in both directions by a current in the wires of the grid induced by the component of the field parallel to the wires. The reflected wave in front of the grid is identical with the scattered field, whereas the transmitted wave behind the grid is equal to the sum of the scattered field and the incident wave (assuming losses in the grid are negligible). Since the scattered field can have a polarization different from the polarization of the primary field, it is possible to produce an elliptically polarized field behind the grid. This fact can be exploited in the construction of antenna systems for circularly or elliptically polarized fields; such antennas have been examined for example by Andreasen [2], who computed the field reflected from two parallel grids being so arranged that the wires of the first grid formed an angle with the wires of the second grid, and by Aagesen [3], who has investigated the field from a reflector consisting of a grid placed parallel to a metal sheet.

B. Diffraction Pattern

A qualitative description of the diffraction pattern around a grid may easily be given for an electromagnetic as well as for an optical transmission grating, the simple mechanism being that the periodic elements of the grid (current-carrying wires or slits with light) radiate elementary cylindrical waves in all directions. The phase of these radiated secondary waves will increase from one wire or slit to the next with the amount $k_0 d \sin\theta$, where $k_0 = 2\pi/\lambda$ is the propagation constant of the incident wave, d the grid constant and θ the angle of incidence of the primary field (this field is assumed not to change in the direction along the wires or slits, $\phi = 0°$). The secondary waves will arrive with the same phase to planes, the normals of which form the angle ψ_n with the positive normal to the grid, where ψ_n is determined from (see Fig. 2)

$$d \sin\theta - d \sin\psi_n = n\lambda, \qquad n = 0, \pm 1, \pm 2 \cdots . \quad (1)$$

By the positive normal to the grid we understand the normal pointing from the front to the back side of the grid (seen from the generator). The angle ψ_n should be counted negative when the corresponding direction is to the same side of the normal to the grid as the direction of propagation of the incident wave, positive when it is to the opposite side.

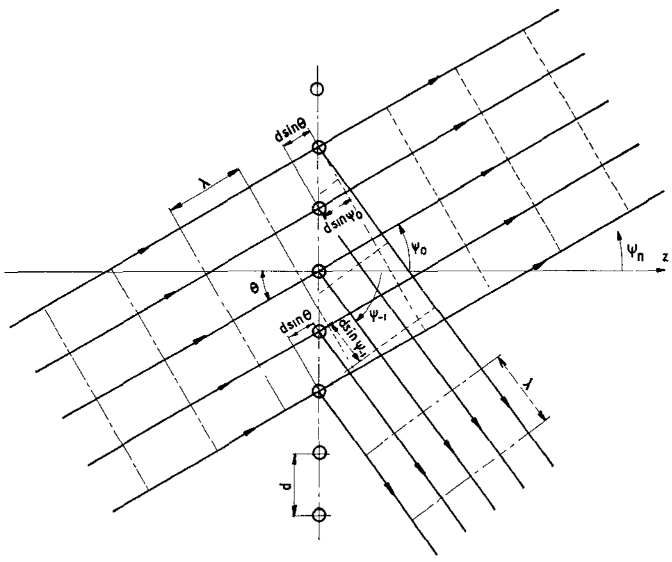

Fig. 2—Diffraction pattern around a grid, side waves are shown for $z>0$ only. The symmetrical waves are present for $z<0$. ($\lambda/d = 1.3$).

The secondary waves propagating from the grid and outwards will form a sequence of plane waves propagating in directions determined by the angles ψ_n.

From (1) we find

$$\sin\psi_n = \sin\theta + n\frac{\lambda}{d}, \qquad n = 0, \pm 1, \pm 2, \cdots . \quad (2)$$

In order to obtain a real solution for ψ_n to this equation we must have

$$\left| \sin\theta + n\frac{\lambda}{d} \right| \leq 1. \quad (3)$$

The number of real solutions ψ_n is therefore finite. For those values of n for which the above inequality is not satisfied we find infinitely many solutions to (2). These solutions, however, are all complex. It may be demonstrated that these complex values of ψ_n correspond to waves propagating along the grid surface, but being attenuated exponentially in the direction perpendicular to the gird. When the wavelength relative to the grid constant increases, more and more of the real side waves will change to attenuated waves. A very illustrative description of the side waves is given by MacFarlane [4].

There will always be at least two real solutions to (2), namely for $n=0$ and $\psi_0 = \theta$ or $\psi_0 = \pi - \theta$, *i.e.*, there will always be a transmitted wave which continues in the direction of the incident wave and a reflected wave with a reflection angle equal to the angle of incidence. If it is desired that only these two zero-order waves shall be present, we must have

$$\frac{\lambda}{d} > 1 - \sin\theta. \tag{4}$$

In optics it is exactly the side waves that are of interest since by using a suitably designed grid we may split a light-wave in waves corresponding to the various colors, as the angle ψ_n becomes different for different wavelengths. This was the purpose of the first practical grids (or diffraction gratings), which were constructed by Fraunhofer in 1823 in connection with his investigations in the field of spectral analysis.

In radio technology it is mostly the directly reflected wave and not the side waves that is of interest, since the grids are often used as a substitute for solid metal plates for reflecting purposes; for example, grid constructions have been used as reflector surfaces in antenna systems and in ground wire systems for antennas. An experimental study of grid reflectors for antennas has been made for example by Gresky [5] and ground wire systems for antennas have been treated for example by Abbott [6], Monteath [7] and by Knudsen and Larsen [8].

However as the lower limit of the wavelengths used in radio technology becomes shorter and approaches the infrared, the diffraction pattern of grids becomes of interest. A spectrometer for millimeter waves has for example been described by Coates [*9], and Klein, *et al.* [*10] describes how it is possible to generate millimeter waves by letting a magnetron oscillate with higher harmonics and then filtering the waves with desired wavelength by using a diffraction grid.

C. Grid Impedance, Transmission and Reflection Coefficients

The purpose of most of the electromagnetic investigations of grids has been to find the complete field distribution around the grid in the case of a plane incident wave. However, some authors have found it practical to characterize the influence of the grid upon the field exclusively by a single quantity, the grid impedance, or by the reflection and the transmission coefficients. These quantities are of most value for the directly transmitted and reflected wave, but they have been defined also in the case where side waves occur.

The grid impedance has been defined in two different ways. Wessel [11] and Hornejäger [12] (see Section III-B) defined their grid impedance Z_w per unit length of wire as the ratio between the electric field strength parallel to the wires and the current of a single wire, and used this definition also when side waves occur. MacFarlane [4] and Wait [13] (see Section III-B) calculated the reflection coefficient of the grid and found that it had the same form as the reflection coefficient for a transmission line shunted with an impedance. Accordingly, they defined their grid impedance Z_g as the shunt impedance of the equivalent transmission line.

The connection between the two impedances is given by

$$Z_w d = \frac{Z_0}{2} + Z_g, \tag{5}$$

where Z_0 is the characteristic impedance of the equivalent transmission line, which in the general case has the form

$$Z_0 = \zeta_0 \frac{\cos\phi}{\cos\theta}, \tag{6}$$

where ζ_0 is the characteristic impedance of free space.

The voltage reflection and transmission coefficients r_v and t_v are defined as the ratio between the field strength of the reflected and the transmitted wave to the field strength of the incoming wave (same polarization).

The power reflection and transmission coefficients r_p and t_p are defined from these quantities by:

$$r_p = |r_v|^2, \tag{7}$$

$$t_p = |t_v|^2. \tag{8}$$

For a lossless grid with no side waves we have

$$r_p = 1 - t_p. \tag{9}$$

For some applications it is sufficient to attribute to the grid surface an infinitely large conductivity in the direction of the wires, and the conductivity zero in the transverse direction. For example, this approximate description has been used in the investigations by Andreasen [2] and by Aagesen [3] referred to above, and it is also used to describe a helix in a traveling-wave tube, the helix being approximated by a tube with an infinitely thin wall having an infinitely large conductivity in the direction of the wires, and the conductivity zero in the transverse direction (see for example Chu and Jackson [14]).

However, in computations regarding ground wire systems the simple description of the impedance is insufficient. In such cases the equivalent shunt impedance Z_g has been used.

The impedance properties of the grid for electromagnetic waves may be utilized technically for example for matching purposes (described for example by Jones and Cohn [15] for a dielectric lense), for filters (for example described by Lewis and Casey [*16]) and for artificial dielectrics (described for example by Kaprielian [17]).

III. The Development of the Theory of Grids

A. Investigations from an Early Period

As was mentioned in Section II-B Fraunhofer was the first physicist to make use of grids. This was done in his experiments in spectral analysis around 1823. Since the purpose of this paper is a description of investigations of the electromagnetic properties of grids, and since the literature on grids published from the time of Fraunhofer up to the end of the century is difficult to trace, in what follows only references will be made to papers published in 1889 or later, 1889 being the year when Hertz [1] showed that grids are of interest also for waves having a wavelength essentially larger than the wavelength of light.

The first attempt to a quantitative explanation of Hertz' experiments was made by J. J. Thomson [18] in 1893. He considered a plane grid of infinite extent having a grid constant smaller than the wavelength and with the polarization of the normally incident wave parallel to the wires of the grid. Thomson found that at a certain distance from the grid the intensity of the reflected wave is the same as in the case of reflection from a metallic surface, but that the phase differs with a quantity depending upon the diameter of the wires and the grid constant.

In 1898 Lamb [19] published a paper in which he computed the reflection and the transmission coefficient for the grid as well in the case when the polarization of the incident wave is parallel to the wires as in the case when the polarization is perpendicular to the wires (perpendicular incidence). These computations were based on the potential and the stream functions for the stationary field around the grid. Lamb investigated partly a grid consisting of strips, and partly a grid consisting of circular rods, the grid constant d being in both cases smaller than the wavelength λ. In contradiction to Thomson he found in accordance with later theories and measurements that the reflection coefficient depends upon the wire diameter as may be seen from the following expression found by Lamb for the power reflection coefficient r_p for a grid consisting of circular rods with radius a.

1) Electric vector parallel to the direction of the wires

$$r_p = \frac{1}{1+\left(\frac{2d}{\lambda}\ln\frac{d}{2\pi a}\right)^2}. \tag{10}$$

2) Electric vector perpendicular to the direction of the wires

$$r_p = \frac{\left(\frac{2\pi^2a^2}{\lambda d}\right)^2}{1+\left(\frac{2\pi^2a^2}{\lambda d}\right)^2}. \tag{11}$$

The last formula shows that when $a \ll d$, the reflection coefficient will be very small, *i.e.*, a wave polarized perpendicular to the direction of the wires passes the grid almost uninfluenced as shown experimentally by Hertz.

In 1906 Schaefer and Laugwitz [20] tried to verify J. J. Thomson's results experimentally (at this time they were not acquainted with Lamb's work). They measured and compared the phase difference occurring in the reflection from a metal plate and in the reflection from a grid, and they found that it did not agree at all with Thomson's results. Whereas Thomson had found a pronounced dependence of the grid constant, Schaefer and Laugwitz's experiments showed that there was practically no dependence.

In 1907 G. H. Thomson [21] also carried out measurements to check the theories of J. J. Thomson as well as Lamb's theories. Using a different method he obtained results which agreed well with Lamb's theory but not with the theory of J. J. Thomson. Through G. H. Thomson's paper, Schaefer and Laugwitz became aware of Lamb's investigation, and their paper from 1906 was followed by a second paper [22] in which they showed that their measurements, too, were in agreement with Lamb's theory.

The influence on the transmission through the grid by the material of the wires in the grid was investigated experimentally by Schaefer and Laugwitz [23] in 1907. At this time there were no theoretical investigations available with which the measurements could be compared, since all theories developed up until that time were based on the assumption that the wires were perfectly conducting. Another assumption upon which all theories known to that time are based is that the radius of the wires is much smaller than the grid constant and the wavelength. However, the smaller the radius is, the larger is the influence of the finite conductivity of the wires. For this reason Schaefer and Laugwitz made measurements on grids with very thin wires of various metals. They measured the power transmission coefficient for these grids and found a certain dependence of the grid material.

At the same time as the above theories for explaining Hertz's grid experiments were developed and measurements made, some papers on optical and acoustical grids were published. A few of these papers will be mentioned here.

In optics there was some interest in the influence of grids on the polarization of light. In fact, as early as 1861 Fizeau had made experiments with polarized light incident upon a screen with slits, and Hertz's experiments with grids revived the interest in this field. In 1911 du Bois and Rubens [24] gave a review of all investigations made in this field up until this time, and new investigations were made by themselves. An interest was manifested in investigating whether the light had the same property as Hertz's radio waves, namely, that the effect of the grid is larger when the incident wave is polarized parallel to the wires than it is when the polarization is in the transverse direction. In 1910 Schaefer and Reiche [25] called the two effects the Hertz effect and the Dubois effect (with reference to a previous work of du Bois), respectively, and they investigated theoretically when these two effects will occur. In their computations they assumed that the wire diameter is small as compared to the wavelength, and that the grid constant is so large as compared to the wavelength that the computation of the field can be made on the basis of the computation of the field for a single cylinder. In a following paper from 1911 Schaefer and Reiche [26] elaborated on their theory, still assuming that the grid constant is much larger than the wavelength so that the mutual influence of the wires can be neglected in computing the field.

In acoustics Lord Rayleigh in particular worked on problems regarding the scattering of sound waves around various types of bodies including grids. His investigations form the basis for Lamb's theory of grids from 1898.

All the viewpoints mentioned so far were united in a very comprehensive theoretical investigation by Ignatowsky [27] from 1914. He was the first to consider an angle of incidence different from zero (the plane of incidence being perpendicular to the wires) and an arbitrary cross section as well as an arbitrary material for the wires. Later authors have usually found Ignatowsky's investigation too general and therefore too difficult to interpret clearly, for which reason his formulas have seldom been used. (However in a paper from 1958 Meecham, *et al.* [28] have made numerical computations based directly on Ignatowsky's formulas.)

In 1914 Arkadiew [29] made some experimental investigations of the reflection from grids consisting of only 4 wires. Both ferromagnetic and nonferromagnetic materials were used, but as was the case for Schaefer and Laugwitz in 1907 no theoretical investigations were available at that time with which to compare the experimental results.

This theory was developed in 1920 by Gans [30] and in an appendix [31] to the first paper in 1921. He considered grids with circular wires the radius of which were much smaller than the grid constant, and this again much smaller than the wavelength. In the first paper he assumed that $\mu_r=1$, but in the second paper he found that the formulas were valid also for $\mu_r \neq 1$. Only considering the case of perpendicular incidence he investigated partly the case where the polarization of the incident wave is transverse to the wires, and partly the case where it is parallel to the wires. In the last mentioned case he found the following expression for the voltage reflection coefficient:

$$r_v = \frac{-1}{1 - i\frac{2d}{\lambda}\left(\ln\frac{d}{2\pi a} + \tau_0\right)}, \tag{12}$$

where

$$\tau_0 = \frac{\mu_r}{k_1 a}\,\frac{J_0(k_1 a)}{J_0'(k_1 a)}. \tag{13}$$

Here $J_0(k_1a)$ and $J_0'(k_1a)$ are the zero order Bessel function and its first derivative of argument (k_1a). The formula (12) is seen to agree with (10) calculated by Lamb, when $\tau_0=0$, which will be the case for perfectly conduct ng wires.

Gans made numerical computations in the case of the electric vector being parallel to the wires using the same constants as were used by Schaefer and Laugwitz in their experimental work from 1907. However, it turned out that his results did not agree at all with the measurements. In 1924 Schaefer [32] found a very simple explanation of this fact, as he showed by repeating the experiments that the discrepancy was due to a printing error in the paper from 1907, where the figures for the reflection and the transmission coefficients were changed. After correction of this error the measurements agreed with the theory.

Arkadiew [33] in 1924, too, compared his experimental results for the nonferromagnetic wires with the theory of Gans and found a rather good agreement. In 1926 [34] he extended Gans's theory to include the case of ferromagnetic wires with a complex permeability and compared the results with his experiments from 1914. He found that the formulas found by Gans are valid also when the permeability of the wire material is complex.

B. Recent Investigations

Since the publications of the investigations mentioned in the last section, the last of these being from 1926, little work seems to have been done in the theory of grids for several years. In 1939 the problem was dealt with again, and since the end of the Second World War several contributions have been given to the theory of grids.

With reference to an experimental investigation by Esau, Ahrens and Kebbel [35], Wessel [11] made in 1939 a calculation of the transmission through a grid, and he investigated theoretically as well as numerically the equivalent grid impedance Z_w (for the definition of this impedance see Section II-C). Wessel based his investigation on the method mentioned in Section II-A using the current in the wires of the grid in his computations. He assumed the wires to have infinite conductivity and a circular cross section the radius of which is much smaller than the wavelength. The direction of propagation of the incident wave was assumed to be perpendicular to the plane of the grid. Wessel considered cases where the grid constant varies from being twice the wavelength to values much smaller than the wavelength. His numerical computations showed good agreement with the corresponding experimental investigations.

For the case $d<\lambda$ (no side waves) Wessel found the following expression for the power reflection coefficient (Durchlässigkeit D defined by Wessel equal to t_p):

$$r_p = \frac{1}{1+\left(\frac{\omega L}{R_\infty}\right)^2}, \tag{14}$$

where

$$Z_w = R_\infty - i\omega L = \frac{\zeta_0}{2d} - i\frac{\zeta_0}{\lambda} \cdot \left\{ \ln\frac{d}{2\pi a} + \sum_{n=1}^{\infty} \left\{ \frac{1}{\sqrt{n^2 - \left(\frac{d}{\lambda}\right)^2}} - \frac{1}{n} \right\} \right\}, \quad (15)$$

which for $d \ll \lambda$ (the sum negligible) agrees with the expression (10) given by Lamb.

For $d > \lambda > d/2$ Wessel found for the equivalent grid impedance

$$Z_w = R - i\omega L = \frac{\zeta_0}{2d}\left\{1 + \frac{2}{\sqrt{1 - \left(\frac{\lambda}{d}\right)^2}}\right\} - i\frac{\zeta_0}{\lambda}\left\{\ln\frac{d}{2\pi a e} + \sum_{n=2}^{\infty}\left\{\frac{1}{\sqrt{n^2 - \left(\frac{d}{\lambda}\right)^2}} - \frac{1}{n}\right\}\right\}, \quad (16)$$

i.e., the first term in the sum has turned from an imaginary value to a real value corresponding to the first order side wave turning from an attenuated wave to a propagating wave.

In 1946 MacFarlane [4], like Lamb [19], used a quasi-stationary method of computation, the computation of the transmission coefficient and the impedances in the time dependent case being based on the stationary solution for the field around the grid. However, MacFarlane made an extension of Lamb's investigation in that he investigated the case where the primary wave, polarized parallel to the wires, has an arbitrary angle of incidence. MacFarlane found in this case the following expression for the voltage reflection coefficient under the assumption that $a \ll d$ and that the condition (4) (no side waves) is fulfilled

$$r_v = \frac{-1}{1 - i2\frac{Z_g}{Z_0}}, \quad (17)$$

where Z_g and Z_0 are the impedances defined in Section II-*C* given by

$$Z_0 = \frac{\zeta_0}{\cos\theta} \quad (18)$$

$$Z_g = \zeta_0\frac{d}{\lambda}\left[\ln\frac{d}{2\pi a} + F\left(\frac{d}{\lambda}, \theta\right)\right], \quad (19)$$

where F is given by

$$F = \frac{1}{2}\sum_{n=1}^{\infty}\left\{\left[\left(\frac{d}{\lambda}\sin\theta + n\right)^2 - \left(\frac{d}{\lambda}\right)^2\right]^{-1/2} + \left[\left(\frac{d}{\lambda}\sin\theta - n\right)^2 - \left(\frac{d}{\lambda}\right)^2\right]^{-1/2} - \frac{2}{n}\right\}. \quad (20)$$

This expression is seen to agree with the one found by Wessel, when $\theta = 0°$. F may be regarded as a correction term, which is negligible when $d \ll \lambda$. Curves of F as a function of θ with d/λ as a parameter are given by MacFarlane.

MacFarlane [36] utilized in 1946 like Booker [37] in 1947 Babinet's principle for further grid investigations. MacFarlane's investigations were related to diaphragms in transmission lines and waveguides, whereas Booker made a general investigation of the impedance concept.

In 1948 Hornejäger [12] extended Wessel's investigations to include an angle of incidence different from zero (the incident wave being polarized parallel to the wires), and he considered any value of the ratio between the grid constant and the wavelength.

He found for the power transmission quotient of the direct transmitted wave

$$t_p = \frac{\left(\frac{R}{R_\infty} - 1\right)^2 + \left(\frac{\omega L}{R_\infty}\right)^2}{\left(\frac{R}{R_\infty}\right)^2 + \left(\frac{\omega L}{R_\infty}\right)^2}, \quad (21)$$

where R_∞ is the real part of the grid impedance Z_w, when the inequality (4) is fulfilled (no side waves). R_∞ is given by

$$R_\infty = \frac{\zeta_0}{2d\cos\theta}. \quad (22)$$

The quantities R and ωL are real and imaginary parts of the grid impedance Z_w in the general case:

$$Z_w = R - i\omega L = \frac{\zeta_0}{2d}\left[\frac{1}{\cos\theta} + G\right] - i\frac{\zeta_0}{\lambda}\left[\ln\frac{d}{2\pi a} + \frac{\lambda}{2d}M\right], \quad (23)$$

where G and M are expressions which take different forms according to the relative value of λ/d and θ (depending on the number of real side waves). In the case of no side waves [(4) fulfilled] G and M take the values

$$G = 0 \quad (24)$$

$$M = \sum_{n=1}^{\infty}\left\{\left[\left(n\frac{\lambda}{d} + \sin\theta\right)^2 - 1\right]^{-1/2} + \left[\left(n\frac{\lambda}{d} - \sin\theta\right)^2 - 1\right]^{-1/2} - \frac{2}{n}\frac{d}{\lambda}\right\}. \quad (25)$$

In this case the above expression for the transmission coefficient agrees with the reflection coefficient found by MacFarlane.

In 1949 Miles [38] investigated a grid of strips with an arbitrary value of d/λ, but only for perpendicular incidence. By setting up an integral equation and solving it by variational principles he investigated acoustic waves as well as electromagnetic waves polarized parallel to and perpendicularly to the wires. Originally only infinitely thin strips were considered, but later the investigations were extended to strips of finite thickness.

In 1951 Shmoys [39] formulated an integral equation for computing the scattering matrix for an infinitely large grid, consisting of wires having an infinite conductivity and an arbitrary cross section for the case where the plane of incidence for the incident wave is perpendicular to the wires, the direction of polarization of the incident wave being perpendicular to or parallel to the wires. In the last mentioned case he used a variational method and found an approximate expression for the scattering matrix for a grid, the wire radius and the grid constant of which are much smaller than the wavelength.

In the "Waveguide Handbook" from 1951 Marcuvitz [40] presents formulas and curve sheets for the equivalent, normalized parameters for various grid configurations all of which have been found by an integral equation method. In all the cases considered here the plane of incidence is perpendicular to the wires. The following cases have been treated: grids of infinitely thin strips with either the magnetic or the electric field strength parallel to the wires, grids with a circular or a rectangular cross section in the case where the magnetic field strength is parallel to the wires, and grids with elliptical, circular, and rectangular cross section in the case where the electric field strength is parallel to the wires.

In 1951 Grillini [*41] has published results of an experimental investigation of grids of strips.

In 1952 a new extension of Wessel's and Hornejäger's investigations was made by Lewis and Casey [42] who considered the case where the wires have a finite conductivity. They limited their investigation to the case $d<\lambda/2$, thereby excluding the possibility of side waves, and they gave numerical results for the reflection and the transmission coefficients for various values of the conductivity of the wires for the case where the electric field strength is parallel to the wires but where the direction of propagation of the incident wave is arbitrary. They found that the effect of losses in the wires is to increase the grid impedance Z_w as if the single wire internal impedance per unit length Z_i were added in series. Lewis and Casey used the value of the internal wire impedance given by Ramo and Whinnery [43]:

$$Z_i = -\frac{k_1 J_0(k_1 a)}{2\pi a \sigma_1 J_0'(k_1 a)}, \tag{26}$$

where $k_1 = \sqrt{i\omega\mu_1\sigma_1}$, *i.e.*, it is assumed that $\sigma_1 \gg \omega\epsilon_1$. For high frequencies and not too small wire radii this expression reduces to

$$Z_i \cong \sqrt{\frac{\omega\mu_1}{2\sigma_1}}\,\frac{1-i}{2\pi a}. \tag{27}$$

Curves of the real and imaginary part of Z_i have been calculated by Lewis and Casey.

They elaborated on the formula for Z_i and the formulas given by Hornejäger to find explicit expressions for the numerical value and phase angle of the reflection and transmission coefficient.

However, in order to compare the value of the reflection coefficient with the results found by other authors we write down the complex reflection coefficient found by Lewis and Casey [formula (15) in their paper]

$$r_v = \frac{-1}{1 + Z_i' - iX_p'}, \tag{28}$$

where Z_i' is the normalized internal wire impedance

$$Z_i' = \frac{d\cos\theta}{60\pi} Z_i = \frac{2d\cos\theta}{\zeta_0} Z_i, \tag{29}$$

and where

$$X_p' = \frac{2d\cos\theta}{\lambda}\left[\ln\frac{d}{2\pi a} + F\right], \tag{30}$$

F being the correction factor found by MacFarlane. This expression for the reflection coefficient is seen to agree with those found by MacFarlane and Hornejäger, when $Z_i = 0$. It is also seen to agree with the results found by Gans, when $\theta = 0$ and $\lambda \gg d (F=0)$ as we have

$$\tau_0 = i\frac{Z_i\lambda}{\zeta_0}. \tag{31}$$

Using an integral equation method Müller [44] in 1953 investigated the diffraction around a strip grid, and in a following paper [45] from the same year he demonstrated how his results could be used in investigating discontinuities in waveguides.

Extending Wessel's, Hornejäger's, and Lewis' and Casey's investigations to include a completely arbitrary direction of incidence and polarization and omitting any assumptions regarding the material of the grid wires Wait [13] in 1955 made a very general investigation of the diffraction through a grid with circular wires. Wait discussed the case where the grid-constant is much smaller than the wavelength so that no side waves occur and he derived an expression for the equivalent grid impedance defined on the basis of an equivalent transmission line. Wait found the following expression for the voltage reflection coefficient.

$$r_v = \frac{-1}{1 + 2\dfrac{Z_g}{Z_0}}, \tag{32}$$

where Z_g and Z_0 are the impedances defined in Section II-C.

$$Z_0 = \frac{\zeta_0 \cos\phi}{\cos\theta} \tag{33}$$

$$Z_g = -i\frac{d}{\lambda}\zeta_0 \cos^2\phi\left[\ln\frac{d}{2\pi a} + F\left(\frac{d\cos\phi}{\lambda}, \theta\right)\right] + dZ_i, \tag{34}$$

where

$$F\left(\frac{d\cos\phi}{\lambda}, \theta\right) = \frac{1}{2}\sum_{n=1}^{\infty}\left\{\left[\left(n + \frac{d}{\lambda}\cos\phi\sin\theta\right)^2 - \left(\frac{d}{\lambda}\cos\phi\right)^2\right]^{-1/2} + \left[\left(n - \frac{d}{\lambda}\cos\phi\sin\theta\right)^2 - \left(\frac{d}{\lambda}\cos\phi\right)^2\right]^{-1/2} - \frac{2}{n}\right\}. \tag{35}$$

Wait defines the internal wire impedance in accordance with Schelkunoff [46] as

$$Z_i = \frac{\zeta_1 I_0(-ik_1 a)}{2\pi a I_1(-ik_1 a)}, \tag{36}$$

where I_0 and I_1 are modified Bessel functions. This expression is equal to (26) used by Lewis and Casey, when $\sigma_1 \gg \omega\epsilon_1$ so that $k_1 = \sqrt{i\omega\mu_1\sigma_1}$.

Whereas Lewis and Casey found that the internal wire impedance Z_i should be added in series to the grid impedance Z_w, Wait found that dZ_i should be added in series to the equivalent shunt grid impedance Z_g. This is in accordance with the relation (5).

It is seen that the term F is the same as the F found by MacFarlane when $\cos\phi = 1$. In fact the curves of F as a function of θ with d/λ as a parameter calculated by MacFarlane may be used also to compute the value of F found by Wait, when the parameter is put equal to $d\cos\phi/\lambda$.

In 1955 and 1956 Yampolskii [*47], [48] published two papers on grid theory investigating the reflection coefficient from a wire grid when the electric vector is parallel to the wires and perpendicular to the wires, respectively. In the last mentioned case he found for the voltage reflection coefficient when $\lambda \gg d \gg a$

$$r_v = \frac{3\pi^2 a^2}{d\lambda}; \tag{37}$$

this expression is seen to agree with (11) found by Lamb except for the factor 3 where Lamb found a factor 2.

In an unpublished thesis Pursley [*49] in 1956 has described microwave and infrared light measurements on grids consisting of wires of aluminum and brass.

An experimental investigation of the transmission through inclined wire gratings including some simplified theoretical considerations was given by Snow [50] in 1956, the main emphasis, however, being on the side wave pattern.

By using a variational method Primich [51] in 1957 computed the field around a grid of strips with a perpendicularly incident primary wave and for arbitrary values of d/λ. He compared his results with corresponding measurements and found a reasonable agreement.

A semi-infinite grid has been investigated by Fel'd [*52] (1958), who found that the difference between the field in the case of the infinite grid and in the case of the semi-infinite grid is vanishing except around the first four conductors of the grid.

As was mentioned above on the basis of Ignatowsky's theory, Meesham, *et al.*, [28] in 1958 made numerical computations of the transmission through a grid with $d/\lambda > 1$ for a perpendicularly incident wave as well in the case where the polarization is parallel to as where it is perpendicular to the wires. The computations have been made on the basis of rather complicated series expressions; for this reason only a limited number of numerical results have been obtained. These results have been compared with Pursley's experimental results, and a reasonable agreement was observed.

In a thesis [53] from 1958 and later in a paper [54] Hansen has used an integral equation method for dealing with the problem of the diffraction of a plane, perpendicularly incident wave through a finite number of infinitely long slits in an infinitely large screen. He gives numerical results for the transmission coefficient in the case where a plane wave polarized in the direction perpendicular to the slits hits a screen with one, two, four or six slits, and he compares these results with the results for strip-grids with infinitely many slits or strips.

In 1959 Decker [55] published results of measurements of the transmission through a grid as a function of the angle of incidence and the frequency. The incident wave was polarized parallel to the wires. Decker compared his results with computations made on the basis of the formulas derived by Wait.

In several papers [56]–[58] (1959) and in a thesis [59] (1960) Særmark has treated the problem of diffraction around a number of strips having the same longitudinal direction but being otherwise arbitrarily oriented with respect to each other. In these investigations he uses a new extension of the addition theorem for Mathieu functions [60]. He also considers the case of several parallel slits in a plane screen. Numerical computations show that when the electric field strength is parallel to the slits, the transmission coefficient for four slits or more does not deviate essentially from the corresponding coefficient for a grid with infinitely many slits (except at resonance $\lambda = d$). Hansen found that this is not the case when the electric field strength is transverse to the slits.

In 1959 Skwirzynski and Thackray [61] have made some calculations closely related to the work by Ignatowsky. General but involved formulas are given for an arbitrary angle of incidence (incoming wave polarized parallel to the wires) and approximate formulas suitable for numerical computations are given in the case of normal incidence for a grid with circular wires. These formulas are valid for $d<\lambda/4$ but no restrictions are placed on d in the lower limit as is the case for all the expressions for reflection coefficients, etc., evaluated by the authors mentioned in this section. Numerical calculations of the transmission coefficient for various values of wire radius and distance between wires are given, the lower value of d being equal to twice the wire radius. These results were compared with experimental results by Goodall and Jackson [62] (1959) and good agreement was obtained.

IV. Some Recent Investigations of Special Grids Configurations

A. Grids with Wires of a Special Material

1) Ferromagnetic Grids: Kozinets [*63] has in 1946 and Epelboim [*64] has in 1947 elaborated on Arkadiew's investigations of ferromagnetic grids.

2) Grid Wires of Two Different Materials: In 1956 Paramonov [65] investigated both theoretically and experimentally the transmission through a grid consisting of circular conducting wires uniformly coated with a dielectric (ice); it turns out that the reflection decreases considerably with increasing thickness of the dielectric coating. The computed curves are in good agreement with the experimental data.

3) Grid Wires with Lumped Elements: A new type of grids was invented by Trentini during the Second World War (published 1953 [66]). He inserted usual network elements periodically along the grid wires whereby he was able to obtain an arbitrary impedance for an incident electromagnetic wave. The first grid of this type was applied as an absorbing coating for metal walls in the meter wave range. Later such grids have found application, for example: as matching elements, in filters, and in electromagnetic lenses.

In 1954 Franz [67] published a theoretical investigation of an idealized model of Trentini's absorption grid where he considered an infinite grid consisting of very thin wires with a uniformly distributed complex impedance placed parallel to a plane sheet with infinite conductivity. His investigation also includes the cases where the conducting sheet is coated with an absorbing or dielectric material.

B. Two or More Parallel Grids

The experimental work by Esau, Ahrens, and Kebbel [35] which was carried out in connection with Wessel's theoretical investigations in 1939 also contained measurements on several parallel grids. Franz [68] first of all treated this case theoretically in 1949. Using a simplified form of Wessel's method of computation he investigated an arbitrary number of infinite grids with the same grid constant placed parallel to each other with parallel wires. He found the transmission coefficient for various systems of grids as a function of the angle of incidence, the distance between neighboring grids, and their position relative to each other. He found a fair agreement with Esau, Ahrens, and Kebbel's measurements.

Lewis and Casey [*16] described in 1951 how two parallel grids may be used as an interference filter for microwaves.

In 1952 Groves [69] investigated theoretically as well as experimentally the transmission through two parallel grids the wires of which form an arbitrary angle with each other. He found the power transmission coefficient as a function of the distance between the grids, the parameters of the grids, and the angle which the grid wires form with each other. Groves limited himself to investigating grids with a grid constant much smaller than the wavelength and to an arbitrarily polarized wave with the direction of propagation being perpendicular to the grids. He based his theoretical investigations on Wessel's work, and he found good agreement between measured and theoretical values.

The above mentioned investigation [66] from 1953 by Trentini also describes measurements on two and three parallel grids. In 1955 Trentini [*70] further investigated the transmission through two parallel grids theoretically as well as experimentally. He was particularly interested in investigating under what circumstances the power transmission was a maximum.

In 1956 Fel'd [*71] made a theoretical investigation of the reflection from and the transmission through two grids when the incident wave was polarized parallel to the wires, whereas the angle of incidence was arbitrary.

In connection with work on antenna systems for circularly or elliptically polarized waves Andreasen [2] in 1956 investigated two parallel wire grids, the wires of which form an aribitrary angle with respect to each other.

Several parallel grids or lattices of parallel, cylindrical, conducting rods are of interest for the construction of artificial dielectrics. Twersky [72]–[75] has in several papers from 1950–52 considered the problem of scattering of waves around an arbitrary number of parallel cylinders. He investigated the accuracy of Schaefer and Reiche's [26] method of computation from 1911 which can be used for optical grids where the distance between the single wires is so large as compared to the wavelength that the mutual influence can be neglected. Twersky first extended this method by taking into account the mutual influence between neighboring cylinders, and he later extended the investigation so that he found a completely general expression for the field around an arbitrary number of parallel cylinders of arbitrary cross section. Sakurai [*76] in 1950 and Kaprielian [17] in 1956 have investigated applications of such systems of cylinders for artificial dielectrics.

C. Grid Parallel to the Plane Interface Between Two Media

In 1954 Wait [77] investigated the reflection from a grid parallel to a conducting plane for an arbitrary angle of incidence for the primary field. In this connection he also considered the possibility of constructing an absorption grid by increasing the resistance of the wires, a problem which has been investigated by Franz [67] in 1954. As has been mentioned above also Aagesen [3] in 1957 treated the problem of a grid parallel to a conducting surface with a view to obtaining a polarization transforming reflector.

In 1957 Wait [78]–[80] treated in three papers the grid problem which is of greatest interest for the theory of ground wire systems, namely the computation of the field around a grid placed parallel to the interface between two dielectric media. Wait shows that under certain circumstances the two media and the grid may be considered equivalent to a composite transmission line being shunted with a certain impedance defined as the equivalent grid impedance. This description is valid for 1) oblique incidence, when the electric vector is always parallel to the wires, 2) normal incidence for any polarization, 3) perfectly reflecting interface for any angle of incidence and polarization, and 4) oblique incidence, when the magnetic vector is always perpendicular to the wires. Wait's impedance formulas for the grid have been derived for a grid placed in front of the interface between the two media, but through a simple interchange of symbols his formulas can also be used in the case most often met with in the investigation of ground wire systems, a grid buried in the ground and parallel to the surface of the ground. A numerical investigation of Wait's formulas has been made by Larsen [81] in 1960.

A special configuration of a grid placed parallel to the interface between two media is a grid situated inside a dielectric slab, which for the case of a normally incident plane wave was investigated by Yampol'skii [*82] in 1958.

Sivov [83] has in 1961 made an investigation of a grid with different dielectric materials on either side of the plane of the grid. Both circular and rectangular cross sections of the wires were treated, the magnetic field strength being parallel to the wires.

V. Conclusion

A review has been given of the literature pertaining to the electrical properties of wire grids. A great many papers have been written on this subject most of them investigating the case of a plane wave propagating towards a plane grid consisting of an infinite number of equal, equidistant and parallel wires. Most papers deal with grids with wires of circular cross section, but also a number of papers describing strip grids have appeared and a few concerning grids with wires of arbitrary cross section.

Various special grid configurations have been treated in the literature too, for example, ferromagnetic wire grids, dielectric coated wire grids, grids with lumped-element wires, more parallel grids with parallel wires and with crossed wires, and grids parallel to conducting and dielectric interfaces.

Papers with the main emphasis on the scattered side waves have not been included in this survey.

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